Sphere Drag in a Viscoplastic Fluid

Benjamin Deglo de Besses
Ajilon Technologies–Cénergys, Immeuble Ferber, 78941 Vélizy Cedex, France

Albert Magnin and Pascal Jay
Laboratoire de Rhéologie, Institut National Polytechnique de Grenoble, CNRS 5520, Université Joseph Fourier Grenoble 1, BP 53, Domaine Universitaire, 38041 Grenoble Cedex 9, France

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It is important to determine the drag forces exerted on a sphere moving in a viscoplastic medium in such industrial applications as the lifting of excavated material in the oil industry or the movement of food suspensions. If there is a sufficient yield stress, plasticity enables the sphere to be immobilized. Knowing this stability condition is useful in designing processes.

The work of Chhabra (1993) contains references for the results obtained from studies on the drag of a sphere in various fluids. It is notable that there are few results for viscoplastic fluids. The forces experienced by a sphere moving in a viscoplastic fluid were studied experimentally by Attapatu et al. (1995) and Hariharaputhiran et al. (1998); analytically by Yoshioka et al. (1971); and numerically by Beris et al. (1985), Blackery and Mitsoulis (1997), Beaulne and Mitsoulis (1997), and Liu et al. (2002). It is clear from these studies that the influence of shear-thinning was not completely taken into account. However, it is also clear that wall slip was never taken into account. The importance of slip phenomena in the flow of viscoplastic fluids was studied by Magnin and Piau (1990) and was reviewed by Barnes (1995). Jossic and Magnin (2001) experimentally demonstrated the effect of adherence conditions on the drag of an obstacle in a viscoplastic fluid.

The present study determines the drag coefficient of a sphere in a Herschel–Bulkley viscoplastic fluid in an infinite medium by means of numerical modeling. The Herschel–Bulkley model is widely used because it allows the existence of a yield stress to be represented by means of a Von Mises criterion and the shear-thinning of the flow equation. Based on a systematic study, a correlation was established for the drag coefficient, taking into account the yield stress value, the shear-thinning index, and the adherence or slip conditions at the sphere wall. The calculations also enabled the dimensions of the sheared zone surrounding the sphere to be determined.

Modeling

The stress tensor deviator $\underline{\underline{\tau}}$ in the Herschel–Bulkley viscoplastic model is a function of the second invariant of the stress tensor deviator τ_{II}, the strain rate tensor $\underline{\underline{D}}$, and its second invariant $\dot{\gamma}$. This behavior law is expressed as

$$\begin{cases} \underline{\underline{\tau}} = 2\left(K\dot{\gamma}^{n-1} + \dfrac{\tau_0}{\dot{\gamma}}\right)\underline{\underline{D}} & \text{if } \tau_{II} > \tau_0 \\ \underline{\underline{D}} = 0 & \text{if } \tau_{II} \leq \tau_0 \end{cases} \tag{1}$$

in which τ_0 is the yield stress, K is the consistency, and n is the behavior index. The Oldroyd number (Od) is used to quantify the relation between the plastic and viscous effects. If d ($d = 2r$) is the diameter of the sphere and U its displacement speed, the Oldroyd number is defined by

$$\mathrm{Od} = \frac{\tau_0}{K(U/d)^n} \tag{2}$$

The calculations were performed with the Polyflow finite-element software from Fluent Inc. (Lebanon, NH). Because inertia was not taken into account, the flow has two symmetries: a symmetry of revolution around an axis running parallel to the flow and passing through the center of the sphere and a symmetry with respect to a plane perpendicular to the flow and passing through the center of the sphere. In the case of slip, the boundary condition is total slip (that is, no shear stress at the

Correspondence concerning this article should be addressed to A. Magnin at Albert.Magnin@ujf-grenoble.fr.

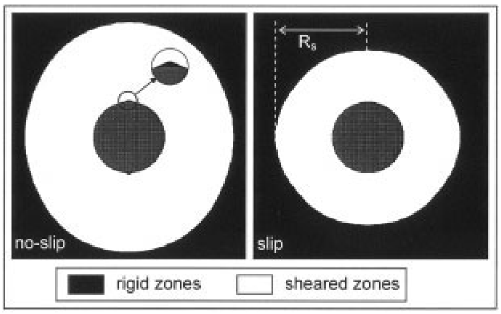

Figure 1. Structure of rigid zones.

Od = 10 and n = 0.34.

wall). In the no-slip case, the boundary conditions is a prescribed translational velocity at the wall U.

The two grids adopted for the calculations consist of quadrilateral elements arranged in blocks. They are similar to those used by Blackery and Mitsoulis (1997). Velocity interpolation within an element is quadratic, whereas pressure interpolation is linear and continuous. These two grids (5200 and 1300 elements) provide drag results that are identical to within 0.1%.

Numerical modeling of the flow of a Herschel–Bulkley fluid involves the regularization introduced by Papanastasiou (1987) with $m = (10^4 \cdot d)/(U \cdot \mathrm{Od})$. The effect of this parameter on the results was discussed extensively by Burgos et al. (1999). A region is considered to be rigid if $\tau_{II} < \tau_0$. Convergence of the stationary regime is obtained by a Newton algorithm combined with Picard iterations (except in the Newtonian case). The iterations cease when the maximum variation in relative velocity is $<10^{-4}$.

Results

Figure 1 shows the distribution of flowing and static and mobile rigid zones. The rigid zone surrounding the sphere is almost an ovoid of little eccentricity. When slip occurs, there is no rigid zone at the stopping points in front of and behind the sphere. Figure 2 shows the change in dimension R_S as a function of the flow parameters. The dimension of the sheared zone in the case of slip is then smaller. The effect of the index n is scarcely felt.

Fluid flow produces a drag force T_z. The drag coefficient expressed in relation to the scale of viscous stresses is defined by

$$C_D = \frac{T_z/\pi r^2}{K(U/d)^n} = \frac{4T_z}{\pi K U^n d^{2-n}} \quad (3)$$

A systematic study of the influence of Od and n showed that a drag coefficient correlation could be expressed by

$$\begin{cases} C_D = [A_1 - A_2(1-n)^2 + A_3(1-n)] \\ \qquad + B_1[1 + B_2(n - B_3)\mathrm{Od}^{-\beta}]\mathrm{Od} \\ \beta = (n + B_4)/B_5 \end{cases} \quad (4)$$

with

	A_1	A_2	A_3	B_1	B_2	B_3	B_4	B_5
Slip	8.000	5.354	8.589	9.731	1.059	0.130	0.196	2.017
No Slip	12.000	16.797	19.976	13.262	1.829	0.151	0.449	2.821

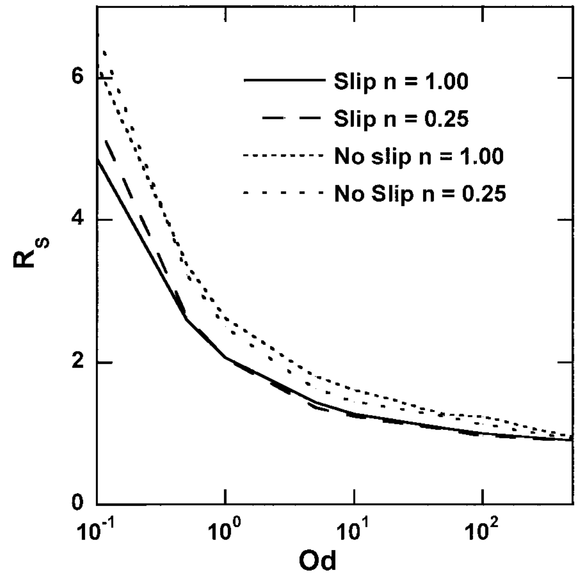

Figure 2. Dimension of the rigid zone.

The relative differences between the drag coefficient values calculated by the correlation and those determined by numerical modeling do not exceed 5%. The values derived from the correlation are valid for $0.25 < n < 1$ and $0 < \mathrm{Od} < 500$.

Figure 3 shows the change in the drag coefficient of a shear-thinning fluid without any yield stress (Od = 0). For the no-slip case, the calculated values are very close to those obtained by numerical modeling by Missirlis et al. (2001) for the motion of a sphere in a tube with an infinite diameter. Figure 4 shows the change in drag coefficient as a function of the Oldroyd number for n = 0.25 and n = 1. The results obtained by Blackery and Mitsoulis (1997) and Liu et al. (2002) in the case of a Bingham fluid (n = 1) are also

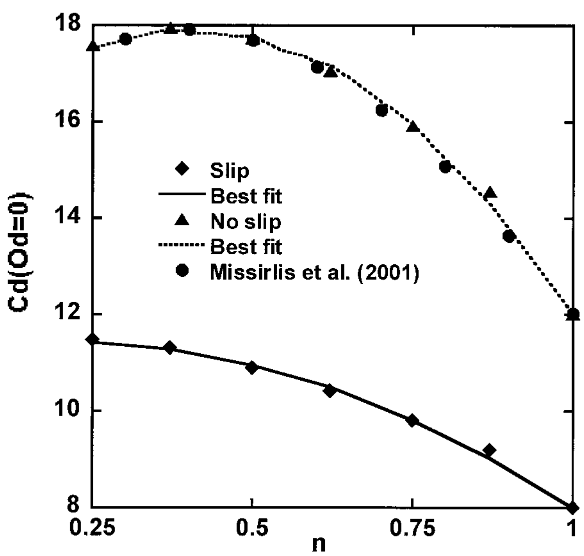

Figure 3. Change in sphere drag coefficient for a shear-thinning fluid without yield stress (Od = 0) and comparison with the results of Missirlis et al. (2001).

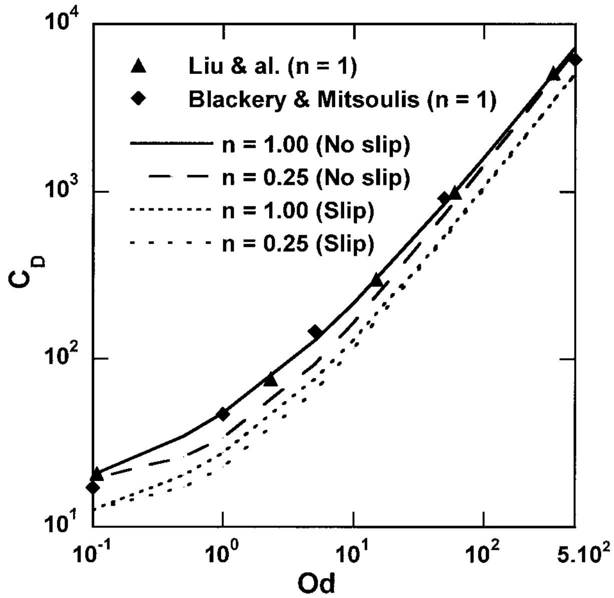

Figure 4. Change in drag coefficient and comparison with the results of Liu et al. (2002) and Blackery and Mitsoulis (1997).

compared. They are in good agreement with those obtained by these authors. Shear-thinning lowers the value of the drag coefficient. The effect of slip at the wall produces similar results.

The results show that slip reduces the drag coefficient by about 30%. This difference can be seen, particularly with respect to the coefficient B_1 corresponding to the drag force on the sphere for an infinite Od number: 9.731 for the slip condition and 13.262 for the no-slip one, respectively. These coefficients can be used to determine the stability condition or mobilization coefficient of the sphere in a viscoplastic fluid. It can be used to estimate the sedimentation or nonsedimentation conditions for a sphere subject to gravity effects. These values can be compared to the value 13.98 obtained by Beris et al. (1985) ($\tau_y^* = 0.143$) with a no-slip condition.

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